

Exercise 78

Find the value of c such that the line $y = \frac{3}{2}x + 6$ is tangent to the curve $y = c\sqrt{x}$.

Solution

We see that the slope of the tangent line is $3/2$. Take the derivative of the equation for the curve.

$$y' = \frac{d}{dx}(c\sqrt{x}) = c \frac{d}{dx}(x^{1/2}) = c \left(\frac{1}{2}x^{-1/2} \right) = \frac{c}{2}x^{-1/2}$$

Set the derivative equal to $3/2$ and solve for x .

$$y' = \frac{c}{2}x^{-1/2} = \frac{3}{2} \quad \rightarrow \quad x = \frac{c^2}{9}$$

This is where the tangent line intersects the curve; that is, the equations that represent them are equal when $x = c^2/9$.

$$\text{At } x = \frac{c^2}{9}: \quad c\sqrt{x} = \frac{3}{2}x + 6 \quad \rightarrow \quad c\sqrt{\frac{c^2}{9}} = \frac{3}{2}\left(\frac{c^2}{9}\right) + 6$$

Solve this equation for c .

$$c = 6$$

Therefore, $y = \frac{3}{2}x + 6$ is tangent to the curve $y = 6\sqrt{x}$ when $x = 4$ as the following figure illustrates.

