## Exercise 78

Find the value of c such that the line  $y = \frac{3}{2}x + 6$  is tangent to the curve  $y = c\sqrt{x}$ .

## Solution

We see that the slope of the tangent line is 3/2. Take the derivative of the equation for the curve.

$$y' = \frac{d}{dx}(c\sqrt{x}) = c\frac{d}{dx}(x^{1/2}) = c\left(\frac{1}{2}x^{-1/2}\right) = \frac{c}{2}x^{-1/2}$$

Set the derivative equal to 3/2 and solve for x.

$$y' = \frac{c}{2}x^{-1/2} = \frac{3}{2} \quad \to \quad x = \frac{c^2}{9}$$

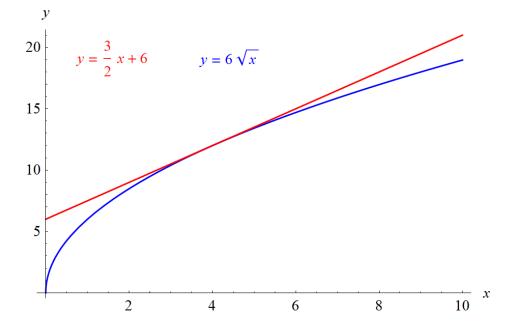
This is where the tangent line intersects the curve; that is, the equations that represent them are equal when  $x = c^2/9$ .

At 
$$x = \frac{c^2}{9}$$
:  $c\sqrt{x} = \frac{3}{2}x + 6 \rightarrow c\sqrt{\frac{c^2}{9}} = \frac{3}{2}\left(\frac{c^2}{9}\right) + 6$ 

Solve this equation for c.

c = 6

Therefore,  $y = \frac{3}{2}x + 6$  is tangent to the curve  $y = 6\sqrt{x}$  when x = 4 as the following figure illustrates.



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